DTU Library

Stochastic Wave Dynamics and Uncertainty Quantification

Engsig-Karup, Allan Peter; Bigoni, Daniele; Glimberg, Stefan Lemvig

Publication date:
2013

Document Version
Publisher's PDF, also known as Version of record

Citation (APA):
Engsig-Karup, A. P., Bigoni, D., & Glimberg, S. L. (2013). *Stochastic Wave Dynamics and Uncertainty Quantification*. Poster session presented at 38th Woudschoten Conference, Zeist, Netherlands.

DTU Compute
Department of Applied Mathematics and Computer Science

GPUlab DTU Informatics

DTU

Stochastic Wave Dynamics and Uncertainty Quantification

Allan P. Engsig-Karup, Daniele Bigoni and Stefan L. Glimberg
Technical University of Denmark (DTU)

Motivation

To address challenges in reliable prediction of extreme events for the design and safe operation of marine systems, simulation-based engineering tools can be used. Such tools are increasingly cost-efficient and can be used to quantify uncertainties and evaluate impact hereof in critical engineering design problems where measurements are infeasible, impractical or too costly.

To characterise uncertainties in wave dynamics and wave-structure responses our objective is to consider modern spectral techniques for uncertainty quantification to describe stochastic properties as accurately as possible in practical times. The spectral techniques provides the basis for meeting the accuracy requirement since these techniques may achieve much faster convergence rates than conventional techniques. To make our approaches practical we seek to combine knowledge in modern algorithms and many-core hardware technologies in a framework to enable efficient stochastic hydrodynamics calculations. Our scope is relevant for computationally intensive (fx. large scale problems) where many simulations are intractable by conventional techniques and approaches.

Case studies

In a first preliminary step, we revisit some classical benchmarks for applications and investigate feasibility of using spectral techniques for quantification of uncertainty in wave dynamics. Possible uncertainty sources are assumed to be

- Boundary conditions (wave generation signal, e.g. wave period and wave amplitude).
- Bathymetry function (sea bed), $h(\mathrm{x},\omega)$.

Contributions

- Stochastic formulation of fully nonlinear and dispersive wave equations.
- Investigation of spectral uncertainty quantification techniques.
- Integration of our research in fast hydrodynamics simulations with spectral uncertainty quantification techniques.

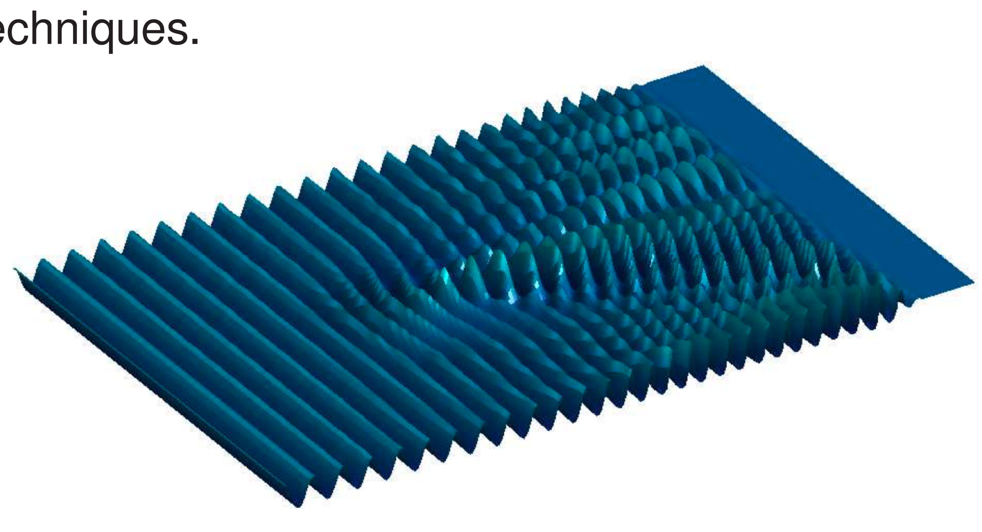


Figure 1 : *Snap shot of deterministic wave field produced by Berkhoff experiment.*

Deterministic formulation

To describe nonbreaking irrotational ocean waves, a fully nonlinear and dispersive water wave is used. Dynamic and kinematic free surface boundary conditions are

$$\partial_t\zeta = -\nabla\zeta\cdot\nabla\tilde{\phi} + \tilde{w}(1+\nabla\zeta\cdot\nabla\zeta),$$
$$\partial_t\tilde{\phi} = -g\zeta - \frac{1}{2}\left(\nabla\tilde{\phi}\cdot\nabla\tilde{\phi} - \tilde{w}^2(1+\nabla\zeta\cdot\nabla\zeta)\right),$$

where $\tilde{\phi} = \phi(x,\zeta,t)$, $\zeta(\mathrm{x},t)$ and $\tilde{w} = \partial_z\phi|_{z=\zeta}$ are free surface quantities and g gravitational acceleration. A Laplace problem needs to be solved

$$\phi = \tilde{\phi}, \quad z = \zeta(\mathrm{x},t),$$
$$\nabla^2\phi + \partial_{zz}\phi = 0, \quad -h \le z < \zeta(\mathrm{x},t),$$
$$\partial_z\phi + \nabla h\cdot\nabla\phi = 0, \quad z = -h.$$

from which closure is obtained by $(\mathrm{u},w) = (\nabla,\partial_z)\phi$.

Stochastic formulation

To enable quantification of uncertainties, a stochastic formulation is obtained by introducing $\omega \in \Omega$ as random input of the system defined in the probability space $(\Omega,\mathcal{F},\mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is a σ-field and $\mathcal{P}$ is a probability measure. This makes the solution a random quantity $\zeta(\mathrm{x},t,\omega) : \bar{D}^{FS}\times[0,T]\times\Omega\to\mathbb{R}$ and $\phi(\mathrm{x},t,\omega) : \bar{D}\times[0,T]\times\Omega\to\mathbb{R}$. $\bar{D}$ is the closed spatial domain volume with FS indicating the restriction to the free surface, $\bar{D} = \{\mathrm{x}|\mathrm{x}\in\xi\}$. A parametrization of the stochastic model is required in order to solve it numerically. A random vector $\mathrm{Z}:\Omega\to\mathbb{R}^d$, is introduced to characterise random inputs, where $d\ge 1$ the stochastic dimension. The stochastic formulation is then

$$\partial_t\zeta(\mathrm{x},t,\mathrm{Z}) = -\nabla\zeta\cdot\nabla\tilde{\phi} + \tilde{w}(1+\nabla\zeta\cdot\nabla\zeta),$$
$$\partial_t\tilde{\phi}(\mathrm{x},t,\mathrm{Z}) = -g\zeta - \frac{1}{2}\left(\nabla\tilde{\phi}\cdot\nabla\tilde{\phi} - \tilde{w}^2(1+\nabla\zeta\cdot\nabla\zeta)\right),$$

where for any (random) sea state, the Laplace problem is fulfilled to obtain closure. This is a stochastic system where unknown variables are random processes.

Generalized Polynomial Chaos

We use generalized Polynomial Chaos (gPC) to create surrogate functions of stochastic variables of the form

$$f(\mathrm{z}) \approx \tilde{f}(\mathrm{z}) = P_N f(\mathrm{z}) = \sum_{i=0}^{N}\hat{f}_i\Phi_i(\mathrm{z}), \quad \hat{f}_i = \frac{(f,\Phi_i)_{\rho_z}}{\|\Phi_i\|_{\rho_z}}.$$

From these, cheap and exponentially accurate statistics for uncertainty quantification can be obtained

$$\mathrm{E}[f(\mathrm{z})] \approx \mathrm{E}[\tilde{f}(\mathrm{z})] = \hat{f}_0,$$
$$\mathrm{Var}[f(\mathrm{z})] \approx \mathrm{Var}[\tilde{f}(\mathrm{z})] = \sum_{i=1}^{N}\hat{f}_i^2\,\|\Phi_i\|_{\rho_z}^2.$$

The unknown gPC expansion coefficients are determined from a solution ensemble by forward propagation of uncertainties via a stochastic collocation method.

Massively Parallel Computing

To enable fast analysis and resolution of large maritime areas, we take advantage of distributed massively parallel high-performance and heterogenous computing on modern many-core hardware. A massively parallel solver has been prototyped in our in-house GPULab library and can be used for stochastic wave dynamics calculations. The fast model enable acceleration of sampling-based (non-intrusive) UQ algorithms in the field of study.

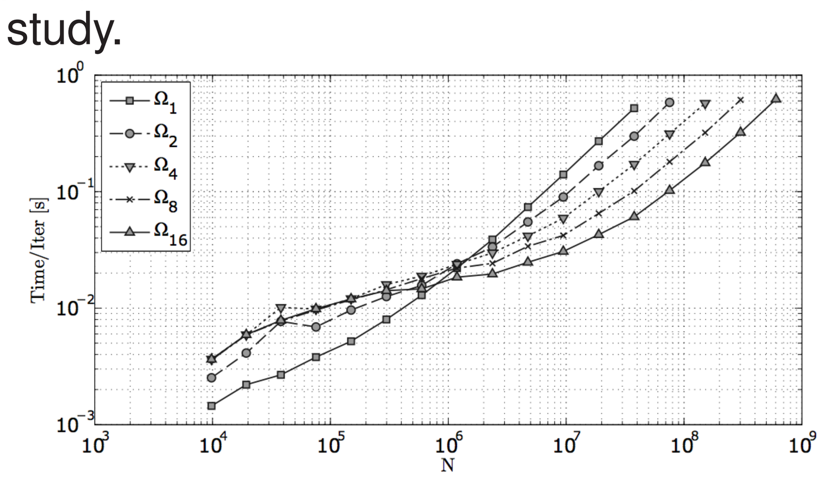


Figure 2 : *Absolute run timings in single precision for heterogenous multi-GPU configurations as a function of number of grid points for iterative PDC method.*

- Efficient multigrid Preconditioned Defect Correction (PDC) method for arbitrary-order discretizations.
- Minimal memory requirements via short recurrence iterative PDC method, matrix-free stencils implementations of sparse operators and single or mixed-precision calculations.
- Fast massively parallel execution on hardware systems of arbitrary size ranging from desktops to super clusters via hybrid MPI-CUDA.
- Fault tolerance and resilience via robust multilevel iterative methods.
- Predictable and scalable performance.

Discretization Methods

To develop a numerical model we use

- Tuneable numerics provides tradeoffs between accuracy and efficiency.
- A flexible-order boundary-fitted Finite Difference Method in space.
- Multigrid Preconditioned Defect Correction (PDC) Method for efficient and scalable iterative solution of Laplace problem every Runge-Kutta stage.
- Data-parallel domain decomposition method implemented for distributed computations.

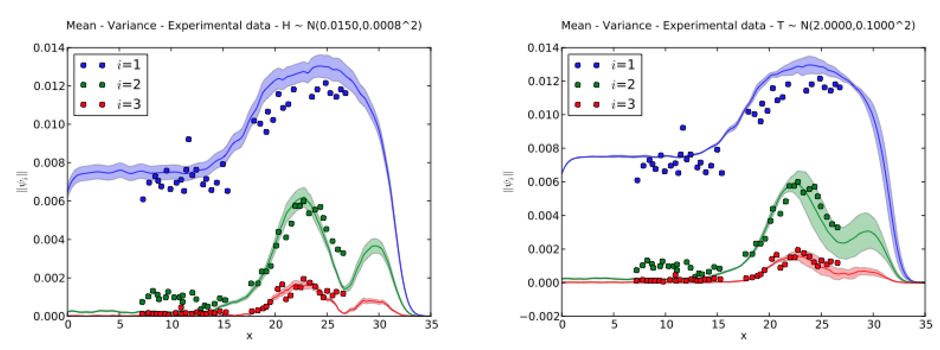
For the time discretization we use

- An explicit fourth-order Runge-Kutta method in time. Due to bounded operator eigenspectra conditional CFL stability without strict step size penalisation due to high-order numerics and/or refined grids.
- A parallel in time (Parareal) discretization to introduce algorithmic concurrency.

For the stochastic discretization we use

- A spectral stochastic collocation method for forward propagation of parametric uncertainty in input data.

Numerical results



(a) Wave Height (b) Wave Period

Figure 3 : *Uncertainty quantification of harmonics contributions to the steady one-wave period solution in Whalin experiment ($T = 2s$) with respect to wave height and wave period. The shaded areas show one standard deviation from the mean (full lines).*

Perspectives

- Promising practical aspects for spectral uncertainty quantification techniques in maritime engineering for low-dimensional stochastic problem.
- Speedup solutions via parallel computations to significantly improve analysis in practical times (but does not resolve curse of dimensionality).
- Stochastic simulation and uncertainty quantification becoming increasingly important for reliable analysis of impact of uncertainties on engineering designs.
- Next steps: better predictions of wave statistics in large (near-costal) finite depths areas and reliable estimations of extreme events.

References

- Engsig-Karup, A. P., Glimberg, L. S., Nielsen, A. S. and Lindberg, O. 2013. Fast hydrodynamics on heterogenous many-core hardware. Part of: Raphäel Couturier (Ed). Designing Scientific Applications on GPUs, 2013, CRC Press / Taylor & Francis Group.
- Engsig-Karup, A. P., Madsen, M. G. and Glimberg, S. L. A massively parallel GPU-accelerated model for analysis of fully nonlinear free surface waves. E-published in *International Journal for Numerical Methods in Fluids*, July, 2011.
- Engsig-Karup, A.P., Bingham, H.B. and Lindberg, O. 2009 An efficient flexible-order model for 3D nonlinear water waves. *Journal of Computational Physics*, 288, pp. 2100–2118.

Contacts: Allan: apek@dtu.dk, Daniele: dabi@dtu.dk, Stefan: slgl@dtu.dk